

# A PROOF OF STRING THEORY FROM A PROOF OF A KALUZA KLEIN THEORY IN 10 DIMENSIONS USING DIFFEO- MORPHISM SYMMETRY • PART 1

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## 1. Abstract.

In December, I wrote many a paper on energy requirements for a proof of string theory and also string theoretic gravitation. As a continuation of those papers, I am writing a paper on a formal proof of string theory. A formal string theory has not been found for many decades. Hence, in this paper I examine a model of string theory where the arbitrary energy field is equivalent to a string manifold.

## 2. Introduction.

Let there be some Energy Field  $\phi$  such that

$$\phi([\phi]_d|\psi(nX_\mu^\mu|\alpha))) = \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \sum_{n \neq 0}^{P_{|\alpha\rangle}} |\psi_n(nX_\mu^\mu|P_{|\alpha\rangle}))$$

Where  $P_{|\alpha\rangle}$  is the state space for all solutions  $|\alpha\rangle$  to the Schrodinger equation in the energy field of the particle and  $[\phi]_d$  being the dependent field at points  $P$  from the particle. We may write that  $P_{|\alpha\rangle}$  is

$$P_{|\alpha\rangle} = \left\{ \bigoplus_{|\alpha\rangle}^{|\beta\rangle} |\alpha\rangle \right\}$$

If we take the Fourier series of  $\phi$ , we will have

$$\begin{aligned} n_0 \phi_0([\phi]_{d_0}|\psi_0(n_0[X_\mu^\mu]_0|\alpha))) \\ \otimes \sum_{n \neq 0} n \phi([\phi]_d \sin ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle))) \\ \otimes \sum_{n \neq 0} n \phi([\phi]_d \cos ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle))) \end{aligned}$$

The model we present is a model where, the energy field of a 10 dimensional open string manifold be equivalent to the Fourier series for the energy field. Thus,

$$\begin{aligned}
& n_0 \phi_0 \left( [\phi]_{d_0} \left| \psi_0 \left( n_0 [X_\mu^\mu]_0 \left| \alpha \right. \right) \right. \right) \\
& \quad \otimes \sum_{n \neq 0} n \phi \left( [\phi]_d \sin ni (k_r x_r^r (\left| \alpha \right.)) - \omega t (\left| \alpha \right.)) \right) \\
& \quad \otimes \sum_{n \neq 0} n \phi \left( [\phi]_d \cos ni (k_r x_r^r (\left| \alpha \right.)) - \omega t (\left| \alpha \right.)) \right) = n \phi (x_0^\mu) + n \phi (\sqrt{2\alpha'} \alpha_0^\mu \tau) \\
& \quad + n \phi (i\sqrt{2\alpha'}) + \phi \left( \sum_{n \neq 0} \frac{1}{n} \alpha_0^\mu e^{-in\tau} \sin n\sigma \right) \otimes \phi \left( \sum_{n \neq 0} \frac{1}{n} \alpha_0^\mu e^{-in\tau} \cos n\sigma \right)
\end{aligned}$$

This is not correct. But we will prove that there exists an equation relating the Fourier series of the field and the field operated the open string expansion,  $X(\sigma, \tau)$ . We prove this by some Wave functional Manipulations, and a proof for the Kazula-Klein theory excluding the Hierarchy Problem in the next part. Our first requirement is to remove the momentum terms due to the fact that our equations must be massless for now.

$$\begin{aligned}
& n_0 \phi_0 \left( [\phi]_{d_0} \left| \psi_0 \left( n_0 [X_\mu^\mu]_0 \left| \alpha \right. \right) \right. \right) \\
& \quad \otimes \sum_{n \neq 0} n \phi \left( [\phi]_d \sin ni (k_r x_r^r (\left| \alpha \right.)) - \omega t (\left| \alpha \right.)) \right) \\
& \quad \otimes \sum_{n \neq 0} n \phi \left( [\phi]_d \cos ni (k_r x_r^r (\left| \alpha \right.)) - \omega t (\left| \alpha \right.)) \right) = n \phi (\sqrt{2\alpha'} \tau) + n \phi (i\sqrt{2\alpha'}) \\
& \quad + n \phi \left( [\phi]_{d_0} \left| \psi_0 \left( n_0 [X_\mu^\mu]_0 \left| \alpha \right. \right) \right. \right) \otimes \phi \left( \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} \sin n\sigma \right) \\
& \quad \otimes \phi \left( \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} \cos n\sigma \right)
\end{aligned}$$

Where we have also replaced  $x_0^\mu$  with  $\left| \psi_0 \left( n_0 [X_\mu^\mu]_0 \left| \alpha \right. \right) \right.$ . These will be our equations that we shall manipulate in the third section.

### 3. General manipulation and a equation for initial terms in the equation

Our goal now is to create a wave function from what we have. The first step is a wave number that is energy bounded.

Lemma 3.1

First let us assume that

$$[n \neq 0] \forall n$$

Now  $k$  is bounded by energy levels. Therefore,

$$\text{Range}(k) = \lim_{\{n \equiv (n_{|\alpha\rangle} \rightarrow n_{|\beta\rangle})\}} k_n$$

Where  $n_{|\alpha\rangle}$  is the ground state and  $n_{|\beta\rangle}$  is the maximum energy state. and we will define now that,

$$\forall k \in k_n$$

End of Lemma.

Lemma 3.2

From Lemma 3.1, we can determine that any  $\psi$  must have  $(k_x, k_x, k_x, \omega)$ . From the Bounded  $k$  equation, we will find that for  $\psi$  and  $k \rightleftharpoons (k_x, k_x, k_x, \omega)$ , we will get

$$\text{Range}(k) = \lim_{\{n \equiv (n_{|\psi(n_{|\alpha\rangle})\rangle} \rightarrow n_{|\psi(n_{|\beta\rangle})\rangle})\}} (k_r, \omega)_n$$

From this we have that any wave function may have  $k$  bounds dependent on its energy bounds.

End of Lemma.

Now we shall focus on a conversion of  $\sin n\sigma \otimes \cos n\sigma$  to the Fourier series of the wave function  $\sin ni(k^r x^r - \omega t) \otimes \cos ni(k^r x^r - \omega t)$ . We are going to prove this using a series of precise alterations. To do that we must create some more lemmas.

Lemma 3.3

To accomplish our objective, we must first understand that

$$\begin{aligned} |\psi_0(nX_\mu^\mu|\alpha)\rangle &= |\psi_0(n_0[X_\mu^\mu]_0|\alpha)\rangle \\ &\otimes \sum_{n \neq 0} \sin ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle)) \otimes \sum_{n \neq 0} \cos ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle)) \end{aligned}$$

$$\begin{aligned}
\therefore e^{ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle))} &= |\psi(nX_\mu^\mu|\alpha)\rangle \\
&= |\psi_0(n_0[X_\mu^\mu]_0|\alpha)\rangle \\
&\otimes \sum_{n \neq 0} \sin ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle)) \otimes \sum_{n \neq 0} \cos ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle))
\end{aligned}$$

Remembering the definition of the wave function. Now, in order to at least prove that

$$\begin{aligned}
&|\psi_0(n_0[X_\mu^\mu]_0|\alpha)\rangle \otimes \sum_{n \neq 0} \sin ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle)) \otimes \sum_{n \neq 0} \cos ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle)) \\
&\leftarrow |\psi_0(n_0[X_\mu^\mu]_0|\alpha)\rangle \otimes \left( \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} \sin n\sigma \right) \otimes \left( \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} \cos n\sigma \right)
\end{aligned}$$

The only method is using the last terms as a "depressed" variation of the wave function Fourier series. For now, we shall ignore the terms  $\frac{1}{n} e^{-in\tau}$ . Now let  $\sigma \rightleftharpoons \sigma(|\alpha\rangle)$ . After this, we will raise the part of the right side of the equation to  $i(\{k_r - \omega\}(|\alpha\rangle))$ . But we must define a map  $\forall \otimes^{-1} \cdot \otimes \times \otimes^{-1} = g_{\mu\nu}$ . Therefore, if we take the product of  $|\psi_0(n_0[X_\mu^\mu]_0|\alpha)\rangle^{-1} \otimes^{-1} |\psi_0(n_0[X_\mu^\mu]_0|\alpha)\rangle \otimes I_{X_\mu^\mu}$ , the result will be

$$|\psi_0(n_0[X_\mu^\mu]_0|\alpha)\rangle^{-1} \otimes^{-1} |\psi_0(n_0[X_\mu^\mu]_0|\alpha)\rangle \otimes I_{X_\mu^\mu} = g_{\mu\nu}$$

On an unrelated note, we remember that  $X_\mu^\mu = \sigma$ . Therefore, we have the opportunity to switch to a decluttered notation.

$$|\psi_n(n_0\sigma_0|\alpha)\rangle^{-1} \otimes^{-1} |\psi_n(n_0\sigma_0|\alpha)\rangle \otimes I_\sigma = g_{\mu\nu}$$

Moving on, we find that

$$\begin{aligned}
&\left( |\psi_0(n_0\sigma_0|\alpha)\rangle^{-1} \otimes^{-1} |\psi_0(n_0\sigma_0|\alpha)\rangle \otimes \left( \sum_{n \neq 0} \sin n\sigma \right) \otimes \left( \sum_{n \neq 0} \cos n\sigma \right) \right)^{i(\{k_r - \omega\}(|\alpha\rangle))} \\
&= e^{n\sigma i(\{k_r - \omega\}(|\alpha\rangle))} \otimes^{-1} |\psi_0(n_0\sigma_0|\alpha)\rangle^{-1} \\
&= e^{ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle))} \otimes^{-1} |\psi_0(n_0\sigma_0|\alpha)\rangle^{-1}
\end{aligned}$$

Now, we just put back the  $|\psi(n_0\sigma_0|\alpha)\rangle$  terms and

$$\begin{aligned}
|\psi(n_0\sigma_0|\alpha)\rangle &\otimes \sum_{n \neq 0} \sin ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle)) \otimes \sum_{n \neq 0} \cos ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle)) \\
&= |\psi_0(n_0\sigma_0|\alpha)\rangle \otimes \left[ \left( \sum_{n \neq 0} \sin n\sigma \right) \otimes \left( \sum_{n \neq 0} \cos n\sigma \right) \right]^{i(\{k_r - \omega\}(|\alpha\rangle))}
\end{aligned}$$

Which we will use as a method to get

$$\begin{aligned}
|\psi_0(n_0[X_\mu^\mu]_0|\alpha)\rangle &\otimes \left( \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} \sin n\sigma \right) \otimes \left( \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} \cos n\sigma \right) \\
&\rightarrow |\psi_0(n_0\sigma_0|\alpha)\rangle \\
&\otimes \sum_{n \neq 0} \sin ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle)) \otimes \sum_{n \neq 0} \cos ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle)) \\
&\equiv |\psi(n_0\sigma_0|\alpha)\rangle \otimes \left( \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} \sin n\sigma \right) \otimes \left( \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} \cos n\sigma \right) \\
&\rightarrow |\psi(n_0\sigma_0|\alpha)\rangle \\
&\otimes \sum_{n \neq 0} \sin ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle)) \otimes \sum_{n \neq 0} \cos ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle)) \equiv
\end{aligned}$$

End of Lemma.

Lemma 3.4

I will develop a final, smaller lemma to get an relation between the Fourier series of the field and the terms  $\left( \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} \sin n\sigma \right) \otimes \left( \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} \cos n\sigma \right)$  from the open string expansion. From the definition of the open string, we encounter the  $\sum_{n \neq 0} \frac{1}{n} e^{-int}$  terms. Let us now, ignoring terms like  $n\phi(\sqrt{2\alpha'}\tau) + n\phi(i\sqrt{2\alpha'})$ , consider,

$$\begin{aligned}
|\psi_n(n_0\sigma_0|\alpha)\rangle &\otimes \left[ \phi \left( \sum_{n \neq 0} \frac{1}{n} e^{-int} [\phi]_d \sin n\sigma |\alpha\rangle \right) \right. \\
&\quad \left. \otimes \phi \left( \sum_{n \neq 0} \frac{1}{n} e^{-int} [\phi]_d \cos n\sigma |\alpha\rangle \right) \right]^{i(\{k_r - \omega\}(|\alpha\rangle))}
\end{aligned}$$

The objective here is to justify the terms  $\sum_{n \neq 0} \frac{1}{n} e^{-int}$  such that,

$$\phi \left( \sum_{n \neq 0} \frac{1}{n} e^{-int} \right) \rightarrow \phi \left( \sum_{n \neq 0} n \right)$$

Realizing that  $\phi = \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu}$ , we will write

$$\begin{aligned}
& n_{A_0} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} |\psi_0(n_0 \sigma_0 | \alpha)\rangle \\
& \otimes \left[ n \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \left( \sum_{n \neq 0} \frac{1}{n} e^{-int} \sin n\sigma | \alpha \rangle \right) \right. \\
& \left. \otimes n \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \left( \sum_{n \neq 0} \frac{1}{n} e^{-int} \cos n\sigma | \alpha \rangle \right) \right]^{i(\{k_r - \omega\}(\alpha))}
\end{aligned}$$

The field of the “depressed wave function” is reduced to for an arbitrary  $n$ , say  $n_A$ , and an arbitrary  $\sigma$ , say  $\sigma_A$ .

$$\begin{aligned}
& n_{A_0} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} |\psi_0(n_{A_0} \sigma_{A_0} | \alpha)\rangle \\
& \otimes \left[ n_A \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \left( \frac{1}{n_A} e^{-in_A t} \sin n\sigma | \alpha \rangle \right) \right. \\
& \left. \otimes n_A \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \left( \frac{1}{n_A} e^{-int} \cos n\sigma | \alpha \rangle \right) \right]^{i(\{k_r - \omega\}(\alpha))}
\end{aligned}$$

In order to obtain that  $\phi \left( \sum_{n \neq 0} \frac{1}{n} e^{-int} \right) \rightarrow \phi(\sum_{n \neq 0} n)$ , we must consider only  $\frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \frac{1}{n_A} e^{-in_A t}$ , and  $\frac{1}{n_A}$  is eliminated and becomes  $n_A$  if we multiply by  $\frac{1}{n_A}^{-2}$ , we have

$$n_A \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} e^{-in_A t}$$

Therefore, the expression is

$$\begin{aligned}
& n_{\mathbb{A}0} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} |\psi_0(n_{\mathbb{A}0} \sigma_{\mathbb{A}0} |\alpha\rangle)\rangle \\
& \otimes \sum_{\{(n_{\mathbb{A}} \neq 0) \in n\} \neq 0} \left[ \frac{1^{-2}}{n_{\mathbb{A}}} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} (\mathfrak{B} e^{-in_{\mathbb{A}} t}) \right. \\
& \otimes \left. \frac{1^{-2}}{n_{\mathbb{A}}} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \left( \frac{d\mathfrak{B} e^{-in_{\mathbb{A}} t}}{n_{\mathbb{A}} \sigma_{\mathbb{A}} d|\alpha\rangle} \right) \right]^{i(\{k_r - \omega\}(\alpha))} \\
& = n_{\mathbb{A}0} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} |\psi_0(n_0 \sigma_0 |\alpha\rangle)\rangle \\
& \otimes \left[ \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \left( \sum_{n \neq 0} n_{\mathbb{A}} e^{-int} \sin n\sigma |\alpha\rangle \right) \right. \\
& \otimes \left. \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \left( \sum_{n \neq 0} n_{\mathbb{A}} e^{-int} \cos n\sigma |\alpha\rangle \right) \right]^{i(\{k_r - \omega\}(\alpha))}
\end{aligned}$$

Where  $\mathfrak{B} = \frac{1}{n_{\mathbb{A}}} \sin n\sigma |\alpha\rangle$ . It is now of upmost importance to multiply both terms  $\frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} (\sum_{n \neq 0} n_{\mathbb{A}} e^{-int} \sin n\sigma |\alpha\rangle)$   $\frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} (\sum_{n \neq 0} n_{\mathbb{A}} e^{-int} \cos n\sigma |\alpha\rangle)$  by  $\sum_{n \neq 0} e^{int}$ .

$$\begin{aligned}
& n_{\mathbb{A}0} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} |\psi_0(n_{\mathbb{A}0} \sigma_{\mathbb{A}0} |\alpha\rangle)\rangle \\
& \otimes \sum_{\{(n_{\mathbb{A}} \neq 0) \in n\} \neq 0} \left[ \frac{1^{-2}}{n_{\mathbb{A}}} \sum_{\{(n_{\mathbb{A}} \neq 0) \in n\} \neq 0} e^{int} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} (\mathfrak{B} e^{-in_{\mathbb{A}} t}) \right. \\
& \otimes \left. \frac{1^{-2}}{n_{\mathbb{A}}} \sum_{\{(n_{\mathbb{A}} \neq 0) \in n\} \neq 0} e^{int} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \left( \frac{d\mathfrak{B} e^{-in_{\mathbb{A}} t}}{n_{\mathbb{A}} \sigma_{\mathbb{A}} d|\alpha\rangle} \right) \right]^{i(\{k_r - \omega\}(\alpha))} \\
& = n_{\mathbb{A}0} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} |\psi_0(n_0 \sigma_0 |\alpha\rangle)\rangle \\
& \otimes \left[ \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \sum_{n \neq 0} e^{int} \left( \sum_{n \neq 0} n_{\mathbb{A}} e^{-int} \sin n\sigma |\alpha\rangle \right) \right. \\
& \otimes \left. \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \sum_{n \neq 0} e^{int} \left( \sum_{n \neq 0} n_{\mathbb{A}} e^{-int} \cos n\sigma |\alpha\rangle \right) \right]^{i(\{k_r - \omega\}(\alpha))}
\end{aligned}$$

The implication is that now, the equations become

$$\begin{aligned}
& n_{\mathbb{A}0} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} |\psi_0(n_{\mathbb{A}0} \sigma_{\mathbb{A}0} |\alpha\rangle\rangle \\
& \otimes \sum_{\{(n_A \neq 0) \in n\} \neq 0} \left[ \frac{1^{-2}}{n_A} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \mathfrak{B} \otimes \frac{1^{-2}}{n_A} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \left( \frac{d\mathfrak{B} e^{-in_A t}}{n_A \sigma_{\mathbb{A}0} d|\alpha\rangle} \right) \right]^{i(\{k_r - \omega\}(\alpha))} \\
& = n_{\mathbb{A}0} \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} |\psi_0(n_0 \sigma_0 |\alpha\rangle\rangle \\
& \otimes \left[ \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \left( \sum_{n \neq 0} n_A \sin n \sigma |\alpha\rangle \right) \otimes \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu^\mu} \left( \sum_{n \neq 0} n_A \cos n \sigma |\alpha\rangle \right) \right]^{i(\{k_r - \omega\}(\alpha))}
\end{aligned}$$

Therefore, we will obtain that

$$\begin{aligned}
& n\phi_0([\phi]_{a_0} |\psi_0(n_0 \sigma_0 |\alpha\rangle\rangle) \\
& \otimes \sum_{n \neq 0} n\phi([\phi]_d \sin ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle))) \\
& \otimes \sum_{n \neq 0} n\phi([\phi]_d \cos ni(k_r x_r^r(|\alpha\rangle) - \omega t(|\alpha\rangle))) \\
& = \left[ \frac{1^{-2}}{n} \otimes \frac{1^{-2}}{n} \left\{ |\psi_n(n_0 \sigma_0 |\alpha\rangle)\rangle^{-1} \otimes^{-1} \sum_{\{(n_A \neq 0) \in n\} \neq 0} e^{int} [X(\sigma, \tau)] \right\} \right. \\
& \quad \left. - (\sqrt{2\alpha'} \tau + i\sqrt{2\alpha'}) \right]^{i(\{k_r - \omega\}(\alpha))} \otimes |\psi_n(n_0 \sigma_0 |\alpha\rangle\rangle
\end{aligned}$$

This is of course assuming, that the string is in a 4-dimensional Relativistic Hyperspace. For a six dimensional hyperspace with number of dimensions  $\mathcal{H}$ , we have

$$\begin{aligned}
& n\phi_0([\phi]_{d_0}|\psi_0(n_0\sigma_0|\alpha))) \\
& \otimes \sum_{\{(n_A \neq 0) \in n\} \neq 0} \phi\left(\frac{e^{in\mathcal{H}}}{(\hat{r}, t)}\right) \sum_{n \neq 0} n\phi([\phi]_d \sin ni(k_r x_r^r(|\alpha)) - \omega t(|\alpha))) \\
& \otimes \sum_{\{(n_A \neq 0) \in n\} \neq 0} \phi\left(\frac{e^{in\mathcal{H}}}{(\hat{r}, t)}\right) \sum_{n \neq 0} n\phi([\phi]_d \cos ni(k_r x_r^r(|\alpha)) - \omega t(|\alpha))) \\
& = \sum_{\{(n_A \neq 0) \in n\} \neq 0} \phi\left(\frac{e^{in\mathcal{H}}}{(\hat{r}, t)}\right) \otimes \sum_{\{(n_A \neq 0) \in n\} \neq 0} \phi\left(\frac{e^{in\mathcal{H}}}{(\hat{r}, t)}\right) \left[\frac{1^{-2}}{n}\right. \\
& \left. \otimes \frac{1^{-2}}{n} \left\{ |\psi_n(n_0\sigma_0|\alpha))\rangle^{-1} \otimes^{-1} \sum_{\{(n_A \neq 0) \in n\} \neq 0} e^{int} [X(\sigma, \tau)] \right\} \right. \\
& \left. - (\sqrt{2\alpha'}\tau + i\sqrt{2\alpha'}t) \right]^{i\{(k_r - \omega)\}(|\alpha))} \otimes |\psi_n(n_0\sigma_0|\alpha))\rangle^{i\{(k_r - \omega)\}(|\alpha))}
\end{aligned}$$

Therefore, an incorporation of open string expansion  $X^{i\{(k_r - \omega)\}(|\alpha))}(\sigma, \tau)$  into the field of the Fourier series of  $e^{ni(k_r x_r^r(|\alpha)) - \omega t(|\alpha))}$

End of Lemma.

#### 4. Proof of some Kazuza-Klein theories through Diffeomorphism Symmetries, and other axioms while preserving S-duality.

From the Kazuza-Klein theory<sup>[1]&[2]</sup> we determine that for an arbitrary theory, we find that.

$$\sum_{\mathcal{H}} \sum_{n \neq 0} \phi_n(\hat{r}, t) \left( \frac{e^{in\mathcal{H}}}{(\hat{r}, t)} \right)$$

We will now calculate an inferred metric of the fields in

$$\begin{aligned}
& n\phi_0([\phi]_{d_0}|\psi_0(n_0\sigma_0|\alpha))) \\
& \otimes \sum_{n \neq 0} n\phi([\phi]_d \sin ni(k_r x_r^r(|\alpha)) - \omega t(|\alpha))) \otimes \sum_{n \neq 0} n\phi([\phi]_d \cos ni(k_r x_r^r(|\alpha)) - \omega t(|\alpha)))
\end{aligned}$$

For a field  $\phi_\mu^\mu$ , define

$$\phi^\mu([\phi]_d|\psi_0(n_0\sigma_0|\alpha))) = \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X^\mu} \sum_{n \neq 0}^{P|\alpha)} |\psi_n(nX_\mu^\mu|P|\alpha))\rangle$$

And

$$\phi_\mu([\phi]_d|\psi_0(n_0\sigma_0|\alpha))) = \frac{\partial^\mu D_\mu E}{\partial^\mu D_\mu X_\mu} \sum_{n \neq 0}^{P|\alpha)} |\psi_n(nX_\mu^\mu|P|\alpha))\rangle$$

Therefore, the result of the inferred metric  $d\phi_{\hat{\mu}}^{\hat{\mu}} = h_{d\alpha} = \partial^{\hat{\mu}}\phi_{\hat{\mu}}\partial_{\hat{\mu}}\phi^{\hat{\mu}}$ . Now assume that for the inferred metric  $h_{d\alpha}$  for  $\mathcal{H}$ , we will obtain

$$d\phi_{\mathcal{H}}^{\mathcal{H}} = h_{d\alpha} = \partial^{\mathcal{H}}\phi_{\mathcal{H}}\partial_{\mathcal{H}}\phi^{\mathcal{H}} \\ \mu \rightleftharpoons \mathcal{H}$$

Now, let  $\mathcal{H}_1 = (\hat{u}, \hat{v}, \hat{w})$  and  $\mathcal{H}_2 = (\hat{l}, \hat{m}, \hat{n})$ . We therefore have 3 inferred metrics

$$\left\{ \begin{array}{l} \partial^{\hat{r}}\phi_{\hat{r}}\partial_{\hat{r}}\phi^{\hat{r}} \\ \partial^{\mathcal{H}}\phi_{\mathcal{H}}\partial_{\mathcal{H}}\phi^{\mathcal{H}} \\ \partial^{\mathcal{H}}\phi_{\mathcal{H}}\partial_{\mathcal{H}}\phi^{\mathcal{H}} \end{array} \right.$$

From this, we can derive the Kazula-Klein theories for the 3 inferred metric,

$$\left\{ \begin{array}{l} \sum_{\mathcal{H}} \sum_{n \neq 0} \phi_{\hat{r}_n}^{\hat{r}} \phi_{\hat{r}_n} \left( \frac{e^{in\hat{r}}}{\hat{r}} \right) \\ \sum_{\mathcal{H}_1} \sum_{n \neq 0} \phi_{\hat{r}_n}^{\hat{r}} \phi_{\hat{r}_n}(\hat{r}) \phi_{\mathcal{H}_1 n}^{\mathcal{H}_1} \phi_{\mathcal{H}_1 n} \left( \frac{e^{in\mathcal{H}_1}}{\hat{r}} \right) \\ \sum_{\mathcal{H}_2} \sum_{n \neq 0} \phi_{\hat{r}_n}^{\hat{r}} \phi_{\hat{r}_n}(\hat{r}) \phi_{\mathcal{H}_2 n}^{\mathcal{H}_2} \phi_{\mathcal{H}_2 n} \left( \frac{e^{in\mathcal{H}_2}}{\hat{r}} \right) \\ \sum_{N > 0} \phi(t_N) \end{array} \right.$$

The next operation will be to complete the theorem by proving that the Kazula-Klein theory satisfies Diffeomorphism symmetry, Weyl invariance, and Lorentz invariance while providing another physical proof.

## 5. Concluding Remarks

It is unfortunate that this project cannot be completed due to the lack of time given. But I believe that the work done over the past 1½ months and will be used to great extent in my own personal work in the next year or so. I do hope that you the reader have gained a bounty of knowledge while reading.

## References

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